VCE MATHEMATICAL METHODS UNITS 1 & 2 – ESSENTIAL CAS CALCULATOR SKILLS

INCL. WORKED EXAMPLES & AN END-OF-YEAR SKILLS CHECKLIST

Reference CAS calculator: Texas Instruments TI-Nspire CAS II

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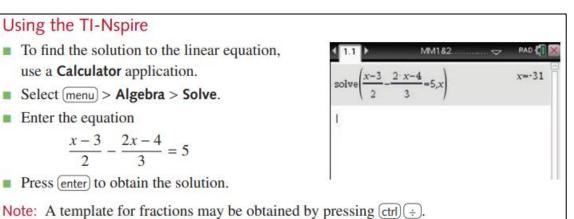
Topic chapters

Chapter 1 – Lines and Linear Relationships

Solving linear equations and inequalities

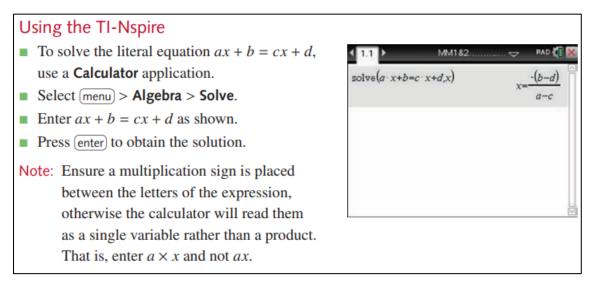
Q: Solve the equation
$$\frac{x-3}{2} - \frac{2x-4}{3} = 5$$
.

A:



Q: Solve the equation ax + b = cx + d for *x*.

A:



Q: Solve the inequality
$$\frac{2x+3}{5} > \frac{3-4x}{3} + 2$$
.

Using the TI-Nspire

The inequality can be solved in a **Calculator** application.

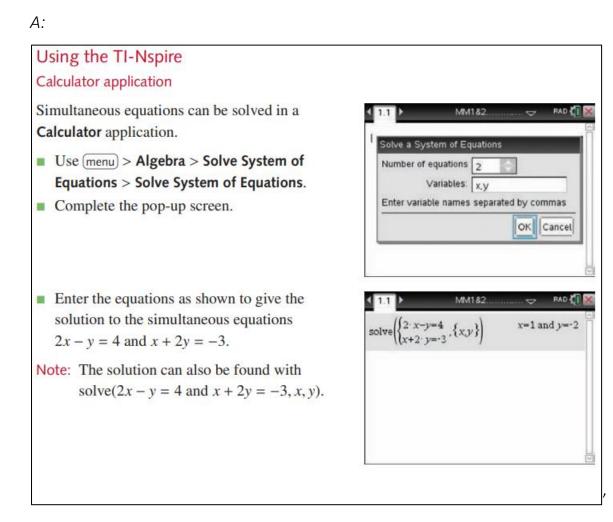
 Choose solve() from the Algebra menu to give the solution to

$$\frac{2x+3}{5} > \frac{3-4x}{3} + 2$$

Note: For the inequality signs template, press (ctrl) =.

Solving simultaneous equations

Q: Solve the equations 2x - y = 4 and x + 2y = -3.



Graphing linear equations

Q: Sketch the graph of 6x + 3y = 9.

A:

RAD

18

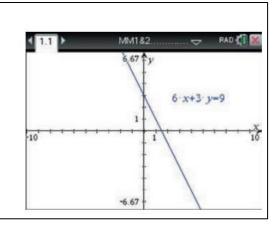
+2,x

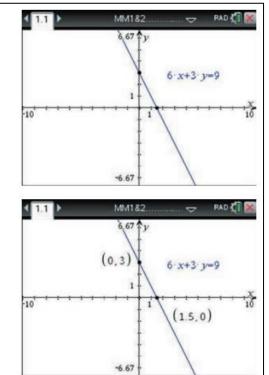
solve

Using the TI-Nspire

To sketch the graph of 6x + 3y = 9:

- Open a Graphs application: press and select the Graphs icon, or use ctrl 1 and select Add Graphs.
- Equations of the form ax + by = c can be entered directly using menu > Graph Entry/Edit > Equation > Line. Enter as 6x + 3y = 9.
- Note: The window settings (<u>menu</u> > Window/Zoom > Window Settings) will have to be changed if the axis intercepts do not appear on the screen.
- The axis intercepts can be found using menu > Geometry > Points & Lines > Intersection Point(s). Select the x-axis and the graph to display the x-axis intercept. Select the y-axis and the graph to display the y-axis intercept.
- To show the coordinates of these points, use menu > Actions > Coordinates and Equations and double click on each of the points.
- Press esc to exit the Coordinates and Equations tool.





Chapter 3 – Quadratic Relationships

Note: The following areas will not be covered in this chapter:

- Solving quadratic equations and inequalities (refer to chapter 1: Solving linear equations and inequalities)
- Solving simultaneous equations involving linear and quadratic equations (refer to chapter 1: Solving simultaneous equations).
- Solving systems of equations involving three or four unknown variables (refer to chapter 1: Solving simultaneous equations).

Expanding factorised polynomial expressions

Q: Expand the expression $(2x - 1)(3x^2 + 2x + 4)$ using a CAS calculator.

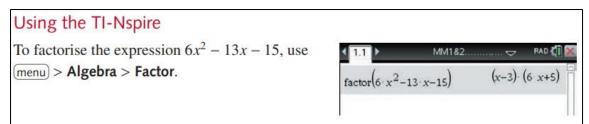
Using the TI-Nspire	
To expand the expression	< 1.1 ► MM1&2
$(2x-1)(3x^2+2x+4)$	$expand((2 \cdot x-1) \cdot (3 \cdot x^2+2 \cdot x+4))$
use menu > Algebra > Expand.	$6 \cdot x^3 + x^2 + 6 \cdot x - 4$
	1

Factorising quadratic expressions

Q: Factorise the expression $6x^2 - 13x - 15$ using a CAS calculator.

А:

A:

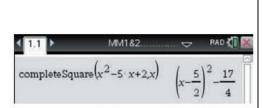


Completing the square for a quadratic expression

Q: Complete the square for the expression $x^2 - 5x + 2$, using a CAS calculator. *A:*

Using the TI-Nspire

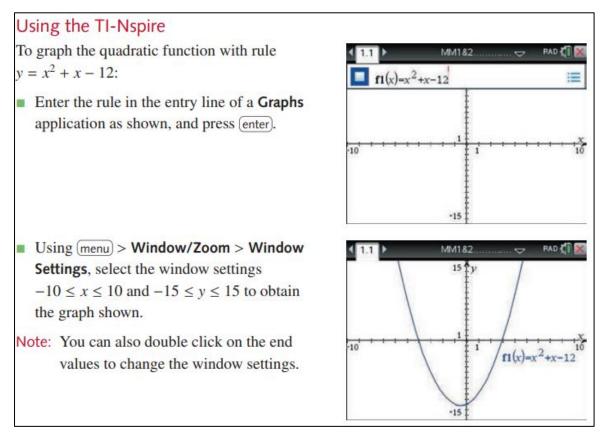
Use (menu) > Algebra > Complete the Square to rearrange the expression $x^2 - 5x + 2$.



Graphing quadratic equations

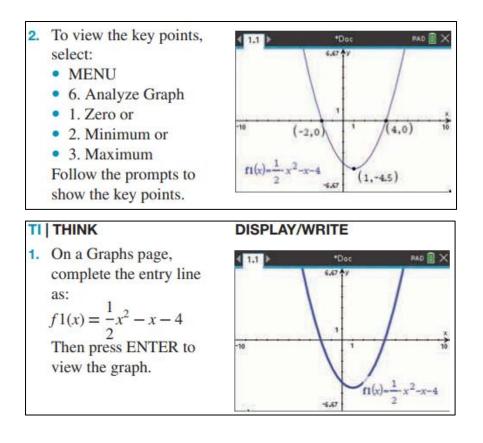
Q: Graph the equation $y = x^2 + x - 12$ in a CAS calculator.

A:



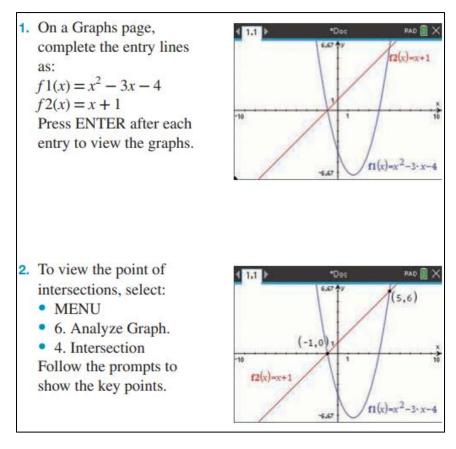
Analysing quadratic graphs

Q: Graph the equation $y = \frac{1}{2}x^2 - x - 4$. Use the graph to determine the coordinates of its x-intercept(s) and turning point.



Finding the points of intersection between a parabola and a line graphically

Q: Graph the equations $y = x^2 - 3x - 4$ and y = x + 1 and find the coordinates of their point(s) of intersection.



Chapter 4: Cubic Polynomials; Chapter 5: Quartic Polynomials

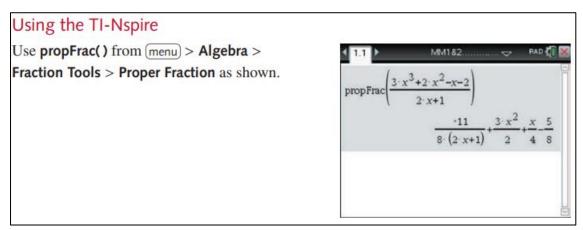
Note: The following areas will not be covered in this chapter:

- Factorising cubic/quartic expressions (refer to chapter 3: Factorising quadratic expressions)
- Solving cubic/quartic equations and inequalities (refer to chapter 1: Solving linear equations and inequalities)
- Graphing cubic/quartic equations (refer to chapter 3: Graphing quadratic equations).

Dividing polynomials

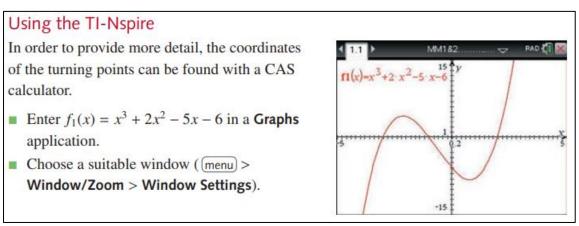
Q: Divide $3x^3 + 2x^2 - x - 2$ by 2x - 1.

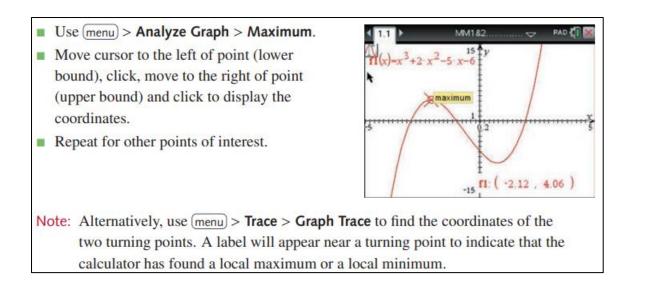
А:



Analysing cubic/quartic graphs

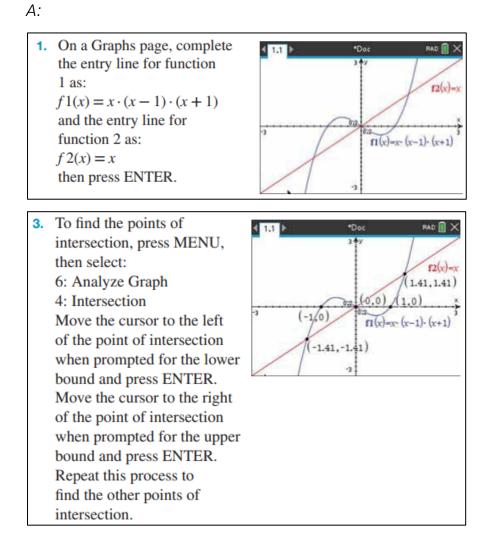
Q: Graph the equation $y = x^3 + 2x^2 - 5x - 6$. Use the graph to determine the coordinates of its maximum and minimum turning points.





Finding the points of intersection between a cubic graph and a line graphically

Q: Graph the equations y = x(x - 1)(x + 1) and y = x and find the coordinates of their points of intersection (exact values are not required).



Chapter 6: Functions and relations

Note: The following areas will not be covered in this chapter:

- Solving equations and inequalities of the form $\frac{1}{x} = a$, $\frac{1}{x^2} = a$, $\sqrt{x} = a$ (refer to chapter 1: Solving linear equations)
- Graphing equations of the form $y = \frac{1}{x'} y = \frac{1}{x^{2'}}$ and $y = \sqrt{x}$ (refer to chapter 3: Graphing quadratic equations).

Defining functions and solving equations in terms of functions

Q: Let f(x) = 2x - 4. Find the values of f(2) and f(t). Solve the equation f(t) = t and the inequality $f(x) \ge x$.

A:

Using the TI-Nspire

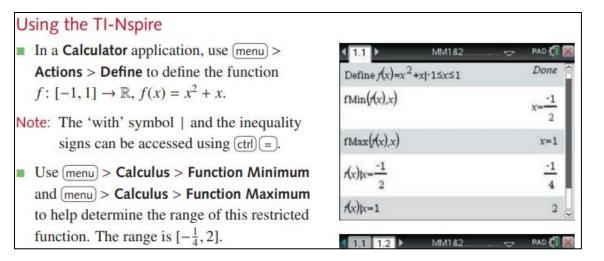
- Use (menu) > Actions > Define to define the function f(x) = 2x 4. Find f(2) and f(t).
- Use (menu) > Algebra > Solve to solve the equation f(t) = t and the inequality $f(x) \ge x$.

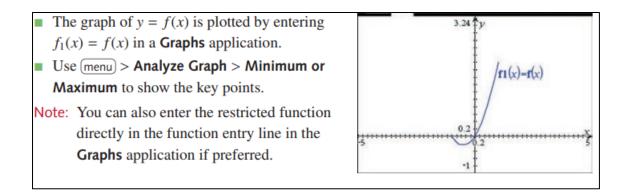
Note: The symbol ≥ can be accessed from the symbols palette ctrl menu or by using ctrl = and selecting ≥.

(1,1)	MM182	RAD 🕻
Define $f(x)=2 \cdot x-4$		Done
1 (2)		0
1 (z)		2· t-4
solve(f(t)=t,t)		t=4
$solve(f(x) \ge x, x)$		x≥4

Restricting functions

Q: Define the function $f: [-1, 1] \to \mathbb{R}$, $f(x) = x^2 + x$. Find the values of x for which the minimum and the maximum values of f occur. Then, show the minimum and maximum of f graphically.





Finding the inverse function of a function

Q: Find the inverse function f^{-1} of the function f(x) = 2x - 3.

A:

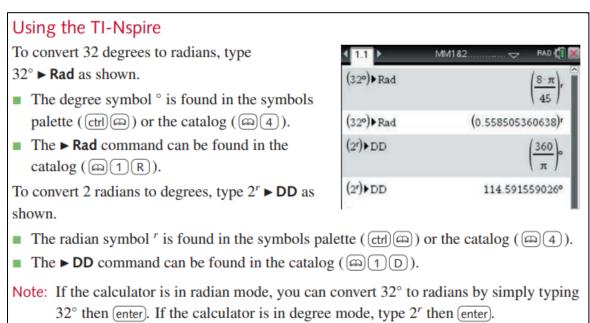
Using the TI-Nspire To find the inverse of the function with rule < 1.1 ▶ MM182 RAD f(x) = 2x - 3, use (menu) > Algebra > Solve. solve(x=2, y=3, y)x+3 Two methods are shown. 2 Define $f(x)=2 \cdot x-3$ Done x=f(y)x=2·y-3 solve(x=f(y),y)x+3 2

Chapter 8: Trigonometric Functions; Chapter 9: Trigonometric Functions and Applications

Converting angles between radians and degrees

Q: Convert **32°** to radians and 2 radians to degrees.

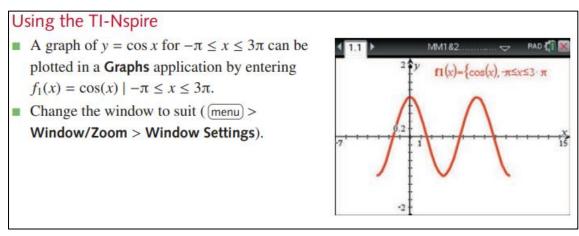
A:



Graphing sine, cosine and tangent functions

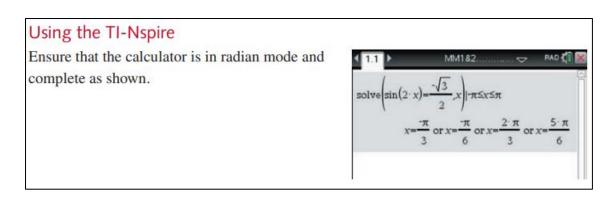
Q: Graph the function y = cosx for $-\pi \le x \le 3\pi$.

A:



Finding a set of solutions to a trigonometric equation with a restricted domain

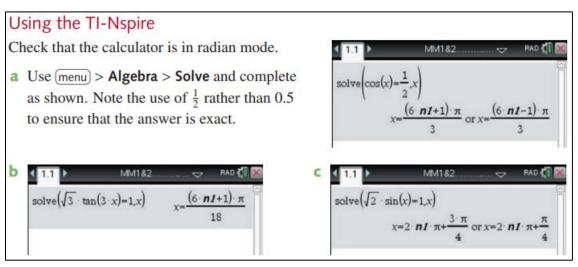
Q: Solve the equation $\sin(3x) = \frac{-\sqrt{3}}{2}$, for $-\pi \le x \le \pi$.



Finding the general solution(s) to a trigonometric equation

Q: Find the general solution(s) to each of the following equations: $\cos(x) = \frac{1}{2}$,

$$\sqrt{3}\tan(3x) = 1, \sqrt{2}\sin(x) = 1.$$



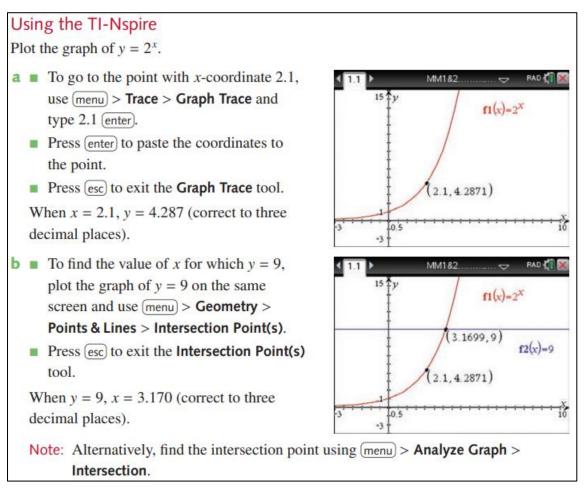
Chapter 10: Exponential Functions and Logarithms

Note: The following areas will not be covered in this chapter:

- Solving exponential and logarithmic equations and inequalities (refer to chapter 1: Solving linear equations and inequalities)
- Graphing exponential and logarithmic equations (refer to chapter 2: Graphing quadratic equations).

Finding the point of intersection between an exponential or a logarithmic graph and a line

Q: Plot the graph of $y = 2^x$ and hence find the value of y when x = 2.1, and the value of x when y = 9.

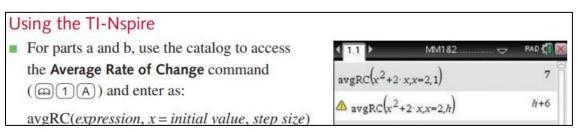


Chapter 11 – Introduction to Differential Calculus: Chapter 12 – Differentiation and Applications

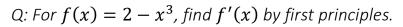
Finding the average rate of change of a polynomial function over a certain domain

Q: For the function $f(x) = x^2 + 2x$, find the average rate of change for $x \in [2,3]$ and $x \in [2,2+h]$.

А:



Finding the derivative of a polynomial function by the limit definition (first principles)

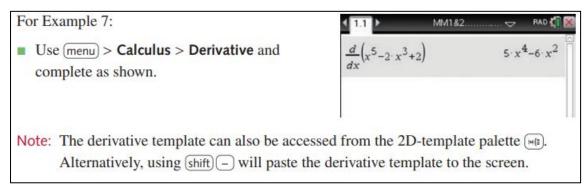


A:

Using the TI-Nspire		
Define $f(x) = 2 - x^3$.	▲ 1.1 ▶ MM182	🗢 🖓 PAD 🕻 🕅 🕅
Use menu > Calculus > Limit or the 2D templete relates (a) and complete	Define $f(x)=2-x^3$	Done
2D-template palette [#f], and complete as shown.		$x^2+3\cdot h\cdot x+h^2$
	$\lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$	$-3 \cdot x^2$
	2	

Finding the derivative of a polynomial function

Q: Differentiate $x^5 - 2x^3 + 2$ *with respect to* x*.*



Finding the value of the derivative of a polynomial function at a certain point

Q: For $f(x) = 3x^3 - 6x^2 + 1$, find f'(1).

A:

For Example 8:	 ▲ 1.1 ► MM182
Define $f(x) = 3x^3 - 6x^2 + 1$.	Define $f(x)=3 \cdot x^3-6 \cdot x^2+1$ Done
Use <u>menu</u> > Calculus > Derivative to differentiate as shown.	$\frac{d}{dx}(f(x)) \qquad \qquad 9 \cdot x^2 - 12 \cdot x$
 To find the value of the derivative at x = 1, use menu > Calculus > Derivative at a Point. 	$\frac{d}{dx}(f(x)) x=1$

Solving equations involving the derivative of a polynomial function

Q: For the curve $y = 4 - x^3$, find the value of y at the point where the gradient of the tangent line is -6.

A:

Using the TI-Nspire		
Define $f(x) = 4 - x^3$.	<1.1 ► MM182	🗢 🖓 RAD 🐔 🕅
Solve the equation $\frac{d}{dx}(f(x)) = -6.$	Define $f(x)=4-x^3$	Done
Substitute in $f(x)$ to find the y-coordinates.	$solve\left(\frac{d}{dx}(f(x))=-6,x\right)$	$x=\sqrt{2}$ or $x=\sqrt{2}$
	$f(\{-\sqrt{2},\sqrt{2}\}) \qquad \{$	$2 \cdot \sqrt{2} + 4 \cdot 4 - 2 \cdot \sqrt{2}$
	1	
		0
	L	

Q: For the function $f(x) = 3x^3 - 4x + 1$, find the coordinates of the stationary points.

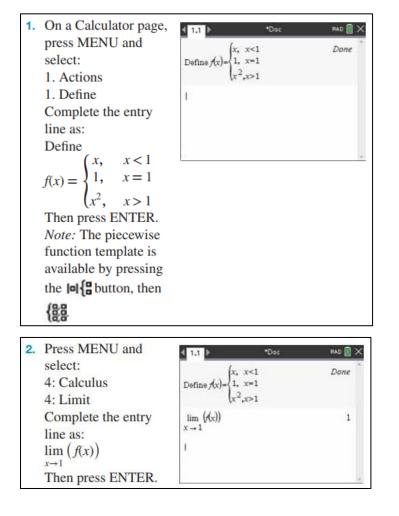
Using the TI-Nspire		
Define the function $f(x) = 3x^3 - 4x + 1$.	▲ 1.1 ▶ MM182	PAD 🕅 🕅
Use (menu) > Algebra > Solve and (menu) > Calculus > Derivative to solve the equation	Define $f(x)=3 \cdot x^3-4 \cdot x+1$	Done
$\frac{d}{dx}(f(x)) = 0$ and determine the coordinates	$\operatorname{solve}\left(\frac{d}{dx}(f(x))=0,x\right)$	$x = \frac{-2}{3}$ or $x = \frac{2}{3}$
of the stationary points.	$f\left(\left\{\frac{-2}{3},\frac{2}{3}\right\}\right)$	$\left\{\frac{25}{9}, \frac{-7}{9}\right\}$
	1	

Finding the limit of a polynomial function or a set of piecewise-defined polynomial functions

Q: Find
$$\lim_{x \to -1} \left(\frac{x^2 - 1}{x + 1}\right)$$
.
A:
1. On a Calculator page, press
MENU and select:
4. Calculus
4. Limit
Complete the entry line as:
 $\lim_{x \to -1} \left(\frac{x^2 - 1}{x + 1}\right)$
Then press ENTER.

Q: For the function below, find $\lim_{x \to 1} f(x)$.

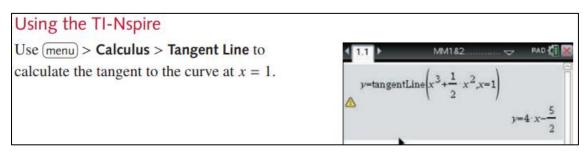
$$f(x) = \begin{cases} x, \ x < 1\\ 1, \ x = 1\\ x^2, x > 1 \end{cases}$$



Finding the equation of the tangent or normal line to a polynomial graph at a certain point

Q: Find the equation of the tangent to the curve $y = x^3 + \frac{1}{2}x^2$ at the point x = 1.

A:



Q: Find the equation of the normal to the curve $y = x^3 - 2x^2$ at the point (1, -1).

A:	
Using the TI-Nspire	
Use $(menu)$ > Calculus > Normal Line to calculate the normal to the curve at the point $(1, -1)$, i.e. when $x = 1$.	$1.1 \qquad $

Chapter 13: Anti-differentiation and Introduction to Differential Calculus

Finding the antiderivative of a polynomial function

Q: Find
$$f(x)$$
 if $f'(x) = x^2 + 2x$ and $f(1) = 1$.

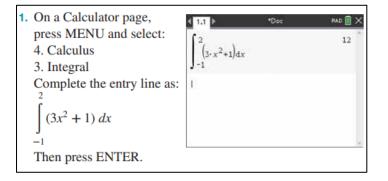
A:

Using the TI-Nspire		
For Example 27a:	<1.1 ► MM182	🗢 🛛 📢 🕅
To find the general antiderivative, define the function f(x) using menu > Calculus > Integral as shown.	Define $f(x) = \int (x^2 + 2 \cdot x) dx + c$ f(x)	Done $\frac{x^3}{x^2+c}$
 Check that c has not been assigned a value. For the specific antiderivative, find the value of c by solving f(1) = 1. 	solve(f(1)=1,c)	$\frac{3}{c=\frac{\cdot 1}{3}}$

Finding the definite integral of a polynomial function

Q: Find the value of $\int_{-1}^{2} (3x^2 + 1) dx$.

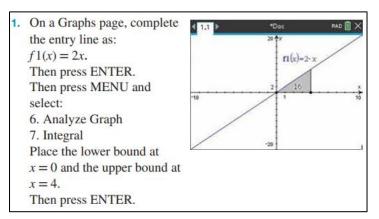
A:



Finding the area under a graph of a polynomial function graphically

Q: Find the area bounded by the line y = 2x, the x-axis and the lines x = 0, x = 4.

А:



Summary of essential skills

Topic chapter	By the end of this chapter, you should be able to do the following using a CAS
1 Lings and Lings an Deletion shine	calculator:
1: Lines and Linear Relationships	 Solve linear equations and inequalities Solve simultaneous equations involving two works over variables
	Solve simultaneous equations involving two unknown variables
2. Quadratia Dalatianshina	Graph linear equations
3: Quadratic Relationships	Expand factorised polynomial expressions
	Factorise quadratic expressions
	 Complete the square for a quadratic expression Color and the square for a quadratic expression
	 Solve quadratic equations and inequalities Solve simultance equations involving linear and modulation equations.
	 Solve simultaneous equations involving linear and quadratic equations Solve systems of a mustice involving three on form under some equilable.
	 Solve systems of equations involving three or four unknown variables
	Graph quadratic equations Analyze guadratic graphs
	 Analyse quadratic graphs Find the preinter of interpreting hot ways a needed by and a line preprint in the second se
4. Cubio Dobro ancialo. E. Quartio	Find the points of intersection between a parabola and a line graphically
4: Cubic Polynomials. 5: Quartic Polynomials	Divide polynomials Sectorize subjective sectorized
Polynomials	Factorise cubic/quartic expressions Calue subjic/guartic expressions
	 Solve cubic/quartic equations and inequalities Create subic (months a subtice)
	Graph cubic/quartic equations Find the preinter of interpretion between a cubic much and a line prechable.
6: Functions and Relations	• Find the points of intersection between a cubic graph and a line graphically
6: Functions and Relations	• Graph equations of the form $y = \frac{1}{x}$, $y = \frac{1}{x^2}$ and $y = \sqrt{x}$.
	 Define functions and solve equations in terms of functions
	Restrict functions
	Find the inverse function of a function
7: Probability	Calculate expressions and solve equations involving the use of factorials
8: Trigonometric Functions, 9:	 Convert angles between radians and degrees
Trigonometric Functions and	Graph sine, cosine and tangent functions
Applications	• Find a set of solutions to a trigonometric equation with a restricted domain
	• Find the general solution(s) to a trigonometric equation
10: Exponential Functions and	 Solve exponential and logarithmic equations and inequalities
Logarithms	Graph exponential and logarithmic equations
	• Find the point of intersection between an exponential or a logarithmic graph
	and a line
11: Introduction to Differential	• Find the average rate of change of a polynomial function over a certain domain
Calculus, 12: Differentiation and	Find the derivative of a polynomial function
Calculus	• Find the value of the derivative of a polynomial function at a certain point
	Solve equations involving the derivative of a polynomial function
	• Find the limit of a polynomial function or a set of piecewise-defined polynomial
	functions
	• Find the equations of the tangent or normal line to a polynomial graph at a
	certain point
13: Anti-differentiation and	Find the antiderivative of a polynomial function
Introduction to Differential	Find the definite integral of a polynomial function
Calculus	Find the area under a graph of a polynomial function graphically
Algorithms and Pseudocode*	Programme codes

*CAS calculator tutorials for the Algorithms and Pseudocode chapter are not included in this document. Refer to the Jacaranda textbook for details.

Appendix: List of useful TI-Nspire CAS calculator shortcuts

Shortcut	Function
Ctrl + A	Select all
Ctrl + C	Сору
Ctrl + H	Find and replace
Ctrl + K	Select page (in split screen)
Ctrl + N	New document
Ctrl + O	Open document
Ctrl + R	Recalculate
Ctrl + S	Save document
Ctrl + V	Paste
Ctrl + W	Close current document
Ctrl + X	Cut
Ctrl + Y	Redo
Ctrl + Z	Undo
Ctrl + 1	Move to end of list/page down
Ctrl + 3	Page down
Ctrl + 4	Merge two pages into split screen
Ctrl + 7	Move to top of list/page up
Ctrl + 6	Convert split screen into two pages
Ctrl + 9	Page up
Ctrl + space	Underscore
Ctrl + tab	Toggle between split screen windows
Ctrl + tab	Toggle between open documents
Shift + (-)	Derivative
Shift + +	Integral
Shift + arrows	Highlight selected text
Shift + esc	Redo